Closing Tue: 9.6

Closing Thu: 9.7(1), 9.7(2)

Closing next Tue: 9.8, 9.9

Exam 1 is Thur, Jan. 31st covers 9.3 - 9.9.

Recall: Finding Derivatives

Step 0: Rewrite powers and simplify.

Step 1: Product, Quotient or Chain?

Step 2: Use appropriate rule, in the middle of that rule you may need to do a derivative (back to step 1)

Entry Task: Find the derivatives of

1. $f(x) = 4x^3(6x^2 + 7)^{10}$
2. $g(x) = \dfrac{3}{\sqrt{x}} + \dfrac{x^2}{5} + \dfrac{x^4}{3x+1}$

[1] PRODUCT RULE!

$f'(x) = \underbrace{4x^3 \cdot 10(6x^2+7)^9 \cdot 12x}_{\text{COMBINE NUMBERS}} + 12x^2(6x^2+7)^{10}$

$f'(x) = 480x^4(6x^2+7)^9 + 12x^2(6x^2+7)^{10}$

ANYTHING IN COMMON? FACTOR?

$f'(x) = 12x^2(6x^2+7)^9\left[40x^2 + (6x^2+7)'\right]$

$f'(x) = 12x^2(6x^2+7)^9(46x^2+7)$

[2] $g(x) = 3x^{-\frac{1}{2}} + \frac{1}{5}x^2 + \frac{x^4}{3x+1}$

$g'(x) = -\frac{3}{2}x^{-3/2} + \frac{2}{5}x + \frac{(3x+1)4x^3 - x^4 \cdot 3}{(3x+1)^2}$

REWRITE

SIMPLIFY

$12x^4 + 4x^3 - 3x^4 = 9x^4 + 4x^3$

$g'(x) = \dfrac{-3}{2x^{3/2}} + \dfrac{2}{5}x + \dfrac{9x^4+4x^3}{(3x+1)^2}$

9.8 The Second Derivative

The *second derivative* is the derivative of the derivative. We denote it

$$f''(x) \quad \text{or} \quad \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

The second derivative represents the rate at which the *rate* of the original quantity is changing.
(*rate of change of rate of change*)
We will interpret more later, for now compute it.

Example:
Assume *x* in seconds and *y* in feet.

$$y = x^3 + 2x \quad \text{feet}$$

$y' = 3x^2 + 2$ $\quad \frac{\text{feet}}{\text{sec}}$ ← velocity

$y'' = 6x$ $\quad \frac{\text{ft/sec}}{\text{sec}}$ ← acceleration

Example: Find $f''(x)$
(use only positive exponents in your final answer)

$$f(x) = \frac{2}{x^3} - \sqrt{x}$$

$f(x) = 2x^{-3} - x^{1/2}$

$f'(x) = -6x^{-4} - \frac{1}{2}x^{-1/2}$

$f''(x) = 24x^{-5} + \frac{1}{4}x^{-3/2}$

$f''(x) = \frac{24}{x^5} + \frac{1}{4x^{3/2}}$

Example: Revenue (in dollars) is given by

$$R(x) = 70x + 0.4x^3$$

if you sell x items.

1. What is marginal revenue (denoted *MR* or $\overline{MR}$) at 10 items?
2. What is the rate of change of marginal revenue when you sell 10 items?

1]

$MR(x) = 70 + 1.2x^2$

$MR(10) = 70 + 1.2(10)^2 = 190 \ \frac{\$}{\text{item}}$

if you go from 10 to 11, then revenue will go up about $\$190$.

2]

$MR'(x) = 2.4x$

$MR'(10) = 2.4(10)$

$= 24 \ \frac{(\$/\text{item})}{\text{item}}$

if you go from 10 to 11 then marginal revenue will go up about $\$24/\text{item}$.